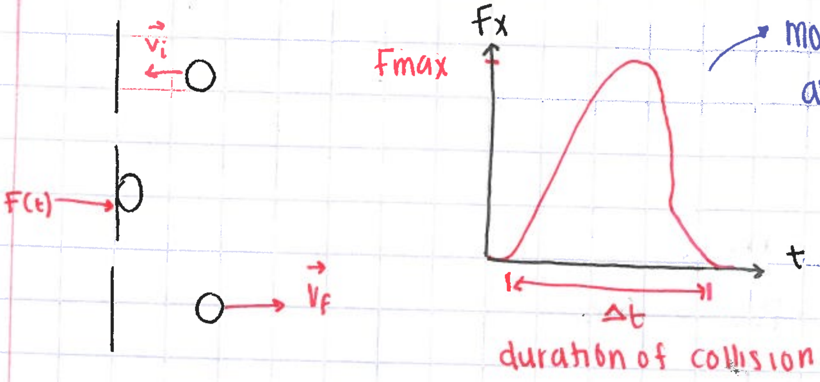
CHAPTER 11: Impulse & Momentum

April 9, 2019

Ex ball hitting wall → sketch F(t) vs t for force on ball



most "real-world" forces are not constant
↓
they start at zero, increase to some maximum value, & then decrease back to zero.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$F_x(t) = m\frac{dV_x}{dt} \rightarrow F_x(t)dt = mdV_x \rightarrow \int_{t_i}^{t_f} F_x(t)\,dt = \int_{V_i}^{V_f} mdV_x$$

$$\rightarrow \int_{t_i}^{t_f} F_x(t)\,dt = m\int_{V_i}^{V_f} dV_x$$

$$= m(V_{fx} - V_{ix})$$

$$\rightarrow \int_{t_i}^{t_f} F_x(t)\,dt = mV_{fx} - mV_{ix}$$

$$P_x = mV_x$$

momentum $\boxed{\vec{p} = m\vec{v}}$

m → mass in kg
v → velocity in m/s

* $\vec{P}$ is a vector that has the same direction as $\vec{v}$

$$[\vec{P}] = \frac{kg\,m}{s}$$

10kg → 2m/s $\quad \vec{P} = 20 \text{ kg m/s } \hat{\imath}$

5kg → 4m/s $\quad \vec{P} = 20 \text{ kgm/s } \hat{\imath}$

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$\vec{P} = m\vec{v}$ ⟶ this is a vector equation

$P_x = mV_x$

$P_y = mV_y$

$P_z = mV_z$

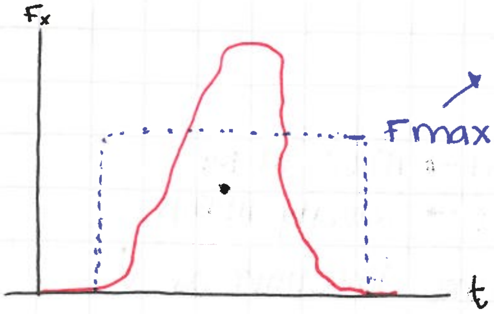
$$\int_{t_i}^{t_f} F_x(t)dt = mV_{fx} - mV_{ix}$$

$$= P_{fx} - P_{ix} = \Delta P_x$$

$$\boxed{\int_{t_i}^{t_f} F_x(t)dt = \Delta P_x}$$ → Impulse-momentum theorem

$$\text{Impulse} = \boxed{J_x = \int_{t_i}^{t_f} F_x(t)dt}$$ → Impulse equals area under the curve of F vs. t.

$[\vec{J}] = N\cdot s$



constant force with same Δt that has same area as the real force.

$$\text{Impulse} \quad J_x = \int_{t_i}^{t_f} F_x(t)dt$$

$= \text{area under curve of } F_x \text{ vs } t$

$= F_{avg,x}\Delta t$

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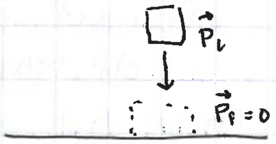
Impulse-momentum theorem:

$$\boxed{\begin{aligned} J_x &= \Delta P_x \\ J_y &= \Delta P_y \\ J_z &= \Delta P_z \end{aligned}}$$

Ex

$F_{ave,x}\,\Delta t = \Delta P_x$

$F_{ave,x} = \dfrac{\Delta P_x}{\Delta t}$



$\Delta P_x = P_{fx} - P_{ix} = 0 - P_{ix}$

In a collision where the object is brought to rest the greater the time of impact (Δt), the smaller the average force:

- air bags
- Crumple zone
- Padding
- dynamic ropes

Problem 11-8

$$\int_{t_i}^{t_f} F_x(t)\,dt = \Delta P_x = P_{fx} - P_{ix}$$

$$P_{fx} = P_{ix} + \int_{t_i}^{t_f} F_x(t)\,dt$$

$$= (2.0\,kg)(1.0\,m/s) + (-2.0\,N)(1.0\,s)$$

$$= 2.0\,kgm/s + (-2.0\,N)(1.0\,s)$$

$$= 0$$

Assume $F = -3.0N$ instead

$P_{fx} = P_{ix} + \int_{t_i}^{t_f} F_x(t)\,dt$

$P_{fx} = -1.0\,kg\,m/s$

$mV_{fx} = -1.0\,kg\,m/s$

$V_{fx} = \dfrac{-1.0\,kg\,m/s}{m} = \dfrac{-1.0\,kg\,m/s}{2.0\,kg}$

$V_f = -0.50\,m/s$

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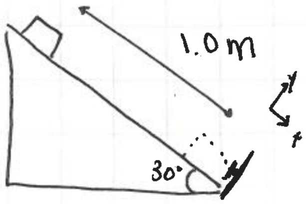
EX PROBLEM 11.40



1) Get speed at bottom of ramp
(from cons. of energy or from eqns. of const $\vec{a}$)

2) Use impulse-momentum to get $\vec{v}$ after collision

3) Get distance up the ramp
(from cons. of energy or from eqns of const. $\vec{a}$)

1) $x_0 = 0m$
$x = 1.0m$
$V_{0x} = 0$ m/s
$V_x = ?$
$a_x = g\sin\theta = 9.80m/s(\sin 30°) = 4.90m/s$
$\Delta t = ?$

$V_x^2 = \overset{=0}{V_{0x}^2} + 2a_x(x - \overset{=0}{x_0})$
$V_x^2 = 2a_x(x)$
$V_x = \sqrt{2a_x(x)}$
$V_x = \sqrt{2(4.90m/s^2)(1.0m)}$
$V_x = 3.13 m/s$

2) $P_{fx} = P_{ix} + \int_{ti}^{tf} F_x(t)dt$

we chose to be pos. down the incline

$= (0.500kg)(3.13m/s) + \frac{(-200N)(26.7\times10^{-3}s)}{2}$

$= -1.164 \text{ kgm/s} = mv_{fx}$

$V_{fx} = \frac{P_{fx}}{m} = \frac{-1.104 \text{ kg m/s}}{0.500kg}$

$V_{fx} = -2.208 m/s$

3) $\frac{1}{2}m\overset{=0}{V_f^2} + mgY_f = \frac{1}{2}mV_i^2 + mg\overset{=0}{Y_i}$
$\cancel{m}gY_f = \frac{1}{2}\cancel{m}V_i^2$
$Y_f = \sqrt{\frac{\frac{1}{2}V_i^2}{g}}$

Answer: $\boxed{0.497m \text{ up ramp}}$

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Impulse-momentum theorem:

$$\begin{tikzcd} & \boxed{J_x = \Delta P_x} \\ \boxed{\vec{J} = \Delta \vec{P}} \arrow[ur] \arrow[dr] & \\ & \boxed{J_y = \Delta P_y} \end{tikzcd}$$

Impulse $\boxed{J_x = \int_{t_i}^{t_f} F_x(t)\,dt}$

$= \boxed{\text{area under the curve of } F_x \text{ vs. } t}$

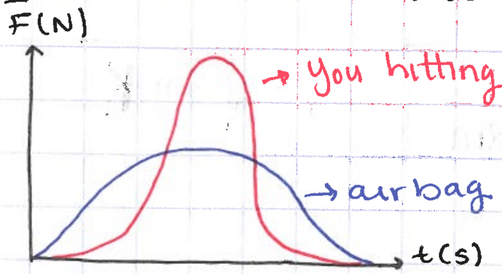
$= \boxed{F_{avg,x}\,\Delta t}$

momentum $\vec{p} = m\vec{v}$ $[\vec{p}] = kg\ m/s$

$$\begin{tikzcd} & \boxed{P_x = mV_x} \\ \boxed{\vec{P} = m\vec{v}} \arrow[ur] \arrow[dr] & \\ & \boxed{P_y = mV_y} \end{tikzcd}$$

$\vec{F}_{avg}\,\Delta t = \Delta\vec{p}$

$\boxed{\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}}$ → In a collision in which an object is brought to rest, $\Delta\vec{p}$ is the same (so J is the same) regardless of Δt. However, greater Δt means smaller $\vec{F}_{avg}$.



$= \vec{F}_{avg}\,\Delta t$

$\vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{p} = \vec{P}_f - \vec{P}_i$

* $\vec{J}$ has the same direction as $\vec{F}_{avg}$ and $\Delta\vec{p}$

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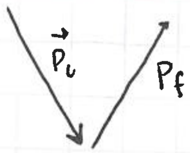

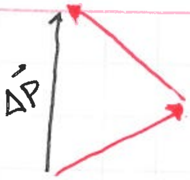

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
$$= \vec{p}_f + (-\vec{p}_i)$$

Ex. 11.4 (Answer: Equal to)
11.5 (Answer: < (less than))

Work-energy principle

$$\boxed{\Delta K = W = \int_{x_i}^{x_f} F_x\,dx}$$

Impulse-momentum principle

$$\boxed{\Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} F_x\,dt}$$

How a $\vec{p}$ and K related?

$\vec{p} = m\vec{v}$ $p = mv$ (magnitude)

$$K = \frac{1}{2}mv^2$$

$$K = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$ $$\boxed{K = \frac{p^2}{2m}}$$ $$p = \sqrt{2mK}$$

Newton's 2nd law

$$\sum\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

ONLY true if
m = constant

$$\boxed{\sum\vec{F} = \frac{d\vec{p}}{dt}}$$ → This is how Newton published his 2nd law

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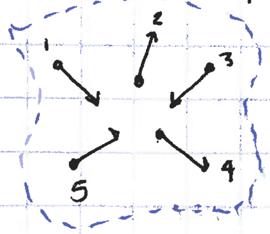
$$\Sigma \vec{F} = \frac{d}{dt}(m\vec{v})$$

$$\Sigma \vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

$$\boxed{\text{Conservation of momentum}}$$

Consider a system of n particles:



Total momentum of a system:

$$\boxed{\vec{P} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n = \sum_{i=1}^{n} \vec{P}_n}$$

$\vec{P}_1 = -10$ kgm/s $\vec{P}_2 = +10$ kgm/s

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = 0$$

IQ: Can a single object have kinetic energy but no momentum? Can a system of two objects have nonzero total kinetic energy but zero total momentum?

c) NO, Yes

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{System} \rightarrow \Sigma \vec{F}_1 + \Sigma \vec{F}_2 + \dots + \Sigma \vec{F}_n = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \dots \frac{d\vec{P}_n}{dt}$$

$$= \frac{d}{dt}(\vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n)$$

$$\boxed{\Sigma \vec{F}_{system} = \frac{d\vec{P}}{dt}}$$

$$P = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n$$

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$$\sum \vec{F}_{system} = \frac{d\vec{P}}{dt}$$

$$\sum \vec{F}_{ext} + \sum \vec{F}_{int} = \frac{d\vec{P}}{dt}$$

$\sum \vec{F}_{int} = 0$ because of Newton's 3rd Law ($\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$)

External Force: Forces the environment exerts on the System.

Internal Forces: Force object within System exerts on each other

$$\boxed{\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}}$$

Conservation of momentum:

if $\sum \vec{F}_{ext} = 0$, then $\frac{d\vec{P}}{dt} = 0$ so $\vec{P}$ = constant

$\vec{P}_f = \vec{P}_i$

* If $\sum \vec{F}_{ext,x} = 0$ $\quad \vec{P}_x$ = constant
* If $\sum \vec{F}_{ext,y} = 0$ $\quad \vec{P}_y$ = constant

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$$\sum \vec{F}_{system} = \frac{d\vec{P}}{dt}$$

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Conservation of momentum:

if $\sum \vec{F}_{ext} = 0$, then $\frac{d\vec{P}}{dt} = 0$ so $\vec{P} = constant$

$\vec{P}_f = \vec{P}_i$

* If $\sum \vec{F}_{ext,x} = 0$ $\vec{P}_x = constant$
* If $\sum \vec{F}_{ext,y} = 0$ $\vec{P}_y = constant$

Recap

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Energy Principle

$$\boxed{\Delta K = W = \int_{x_i}^{x_f} F_x dx}$$

momentum principle

$$\boxed{\Delta P_x = J_x = \int_{t_i}^{t_f} F_x dx}$$

$$\boxed{K = \frac{1}{2} mv^2 = \frac{p^2}{2m}}$$

$$\boxed{P = \sqrt{2mK}}$$

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Newton's 2nd law → $\Sigma \vec{F} = d\vec{p}/dt$

→ For a system of particles:

$\boxed{\Sigma \vec{F}_{system} = \frac{d\vec{p}}{dt}}$ $\vec{p} = \vec{p}_1 + \vec{p}_2 + \cdots + \vec{p}_n$

↓

$\Sigma \vec{F}_{ext} + \Sigma \vec{F}_{int}$

↳ = 0 by Newton's 3rd Law

$\boxed{\Sigma \vec{F}_{ext} = \frac{d\vec{p}}{dt}}$

* conservation of momentum:

$\boxed{\text{If } \Sigma \vec{F}_{ext} = 0,\ \frac{d\vec{p}}{dt} = 0 \rightarrow \vec{p}_f = \vec{p}_i}$

→ If $\Sigma F_{ext,x} = 0$ $\boxed{p_{f,x} = p_{i,x}}$

→ If $\Sigma F_{ext,y} = 0$ $\boxed{p_{f,y} = p_{i,y}}$

↳ your book:

$\frac{d\vec{p}}{dt} = 0$ for an isolated system ($\Sigma F_{ext} = 0$)

11.6 problem.

System → Person + glider

If No air resistance → $\Sigma F_{ext,x} = 0$ $P_{fx} = P_{ix}$

?

$m_{glider} V_{f\,glider} + m_{skydiver} V_{f\,skydiver} = (m_{glider} + m_{skydiver}) V_i$

$m_{glider} = 620$ kg

$m_{skydiver} = 60$kg

$v_i = 30$ m/s

$V_{f,\,skydiver} = 30$ m/s

$$V_{f\,glider} = \frac{(m_{glider} + m_{skydiver}) V_i - m_{skydiver} V_{f\,skydiver}}{m_{glider}}$$

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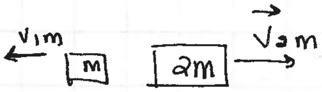
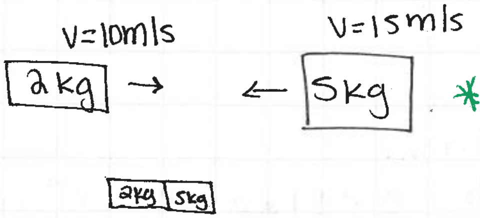
$$V_{glider} = \frac{(680kg)(30m/s) - (60kg)(30m/s)}{620kg}$$

$$= \underline{36m/s}$$

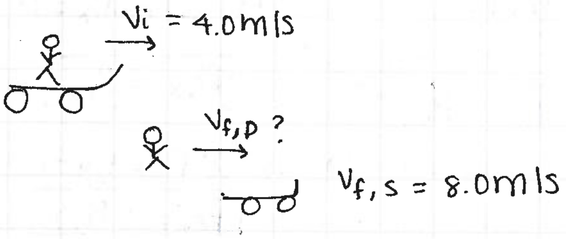
1Q: $\boxed{3m}$





* $V_i = -15m/s$ for the kg mass because it is moving to the left.

Problem 11.28



$m_D = 50kg$

$m_s = 5kg$

System → Dan + Skate board

$\sum F_{ext,} = 0 \qquad P_{f\,x} = P_{i\,x}$

$$m_D V_{f,D} = m_s V_{f,s} = (m_D + m_s) V_i$$

$$V_{f,D} = \frac{(m_D + m_s) V_i - m_s V_{f,s}}{m_D}$$

$$V_{f,D} = \frac{(50kg + 5kg)(4.0m/s) - (5kg)(8.0m/s)}{50kg}$$

$$V_{f,D} = \underline{3.6m/s}$$

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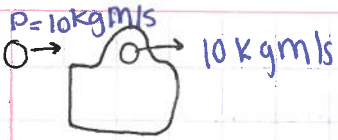

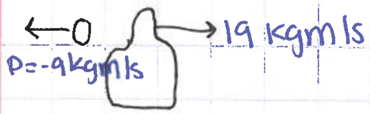

Collisions

*during any collision, you can use conservation of momentum (even if there is friction) as long as you look at $\vec{p}_i$ & $\vec{p}_f$ right before & after the collisions.

(Perfectly) Elastic collision → Kinetic energy is conserved during the collision (No energy lost to head & sound)

↳ Idealization

INELASTIC COLLISION → Kinetic energy is not conserved

Perfectly inelastic collision → Inelastic collision in which objects stick together after the collision.

$$\vec{P}_f = P_i$$

$$\boxed{m_1(V_{fx})_i + m_2(V_{fx})_2 = m_1(V_{ix})_i + m_2(V_{ix})_2}$$

→ Always true for a collision between 2 objects

$$\boxed{(m_1+m_2)V_{fx} = m_1(V_{ix})_1 + m_2(V_{ix})_2}$$

→ only true for completely inelastic collision

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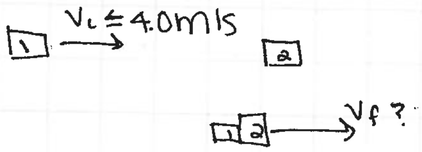
Problem 11-A

1) Use cons. of $\vec{P}$ to get $\vec{V_f}$ after the collision
2) Use cons. of Energy to get Δx

1)



System → both blocks

$$\sum F_{ext,x} = 0 \longrightarrow P_{fx} = P_{ix}$$

$$(m_1+m_2)V_{fx} = m_i V_{1ix} + \overset{=0}{m_2 V_{2ix}}$$

$$V_{fx} = \frac{m_i + V_{iix}}{m_1+m_2} = \frac{(2.0kg)(4.0 m/s)}{(2.0kg+1.0kg} = \underline{2.67 m/s}$$

combined speed of both after the collision.

2) System → both blocks & spring

$$K_i + \overset{=0}{(U_g)_i} + \overset{=0}{(U_s)_i} + \overset{=0}{W_{ext}} = \overset{=0}{K_f} + \overset{=0}{(U_g)_f} + (U_{sp})_f + \overset{=0}{\Delta E_{th}}$$

$$K_i = (U_{sp})_f \longrightarrow \frac{1}{2}(m_1+m_2)V_i^2 = \frac{1}{2}k(\Delta X_f)^2$$

$$(\Delta x_f)^2 = \frac{(m_1+m_2)V_i^2}{k}$$

$$\Delta X_f = \sqrt{\frac{(m_1+m_2)}{k}}V_i$$

$$\Delta X_f = \sqrt{\frac{(3.0kg)}{200 N/m}}(2.67 m/s)$$

$$= \underline{\underline{0.33m}}$$

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$$\boxed{\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} \qquad \vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n}$$

$$\boxed{\text{if } \sum \vec{F}_{ext} = 0 \qquad d\vec{P}/dt = 0 \qquad \vec{P}_f = \vec{P}_i}$$

(Conservation of momentum)

$$\boxed{\text{if } \sum \vec{F}_{ext,x} = 0 \quad \vec{P}_{f,x} = \vec{P}_{i,x}}$$

$$\boxed{\text{if } \sum \vec{F}_{ext,y} = 0 \quad P_{f,y} = P_{i,y}}$$

collision
- inelastic
- Perfectly inelastic
- Perfectly elastic

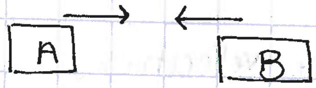
$$\boxed{m_1 (V_{fx})_1 + m_2 (V_{fx})_2 = m_1 (V_{ix})_1 + m_2 (V_{ix})_2}$$

$$\boxed{(m_1 + m_2) V_{fx} = m_1 (V_{ix})_1 + m_2 (V_{ix})_2}$$

Perfectly inelastic collision.

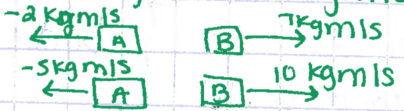
applies even if there is friction/air resistance (so, $\sum \vec{F}_{ext} \neq 0$) as long as we look at $\vec{P}$ right before and right after collision.

10. Blocks A and B have linear momentum with directions as shown and with magnitudes of 9kgm/s and 4kgm/s, respectively. If block A ends up moving to the left is the magnitude of its momentum greater than, less than, or the same as that of block B.

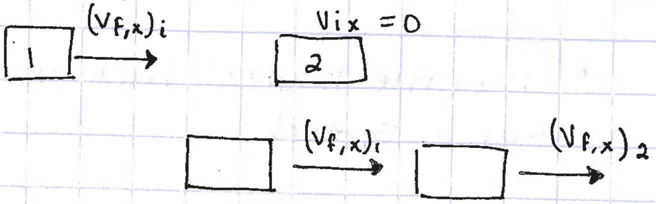


Block A < Block B

Pi = 9kgm/s − 4kgm/s = 5kgm/s



Perfectly Elastic collision (m_2 is initially at rest):



What are $(V_{f,x})_1$ & $(V_{f,x})_2$

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conservation of $\vec{p}$:

$$m_1 (v_{fx})_1 + m_2 (v_{fx})_2 = m_1 (v_{ix})_1 + m_2 \overset{=0}{(v_{ix})_2}$$

Energy conservation:

$$\frac{1}{2} m_1 (v_{f,x})_1^2 + \frac{1}{2} m_2 (v_{f,x})_2^2 = \frac{1}{2} m_1 (v_{i,x})_1^2 + \frac{1}{2} m_2 (v_{ix})_2^2$$

} 2 equations with 2 unknowns

$$\boxed{(v_{f,x})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{i,x})_1}$$

$$\boxed{(v_{f,x})_2 = \frac{2m_1}{m_1 + m_2} (v_{i,x})_1}$$

* Perfectly elastic collision in which m_2 is initially at rest

1 → Projectile
2 → target

3 Special cases

1) $m_1 = m_2$ (Pool player's result)

$$\left.\begin{aligned} (v_{f,x})_1 &= 0 \\ (v_{f,x})_2 &= (v_{i,x})_1 \end{aligned}\right\}$$ bodies exchange velocities

2) massive target ($m_2 \gg m_1$)

$(v_{f,x})_1 \approx -(v_{i,x})_1$ → Projectile bounces back with approx. the same speed

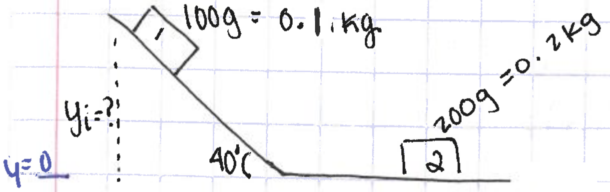
$(v_{f,x})_2 \approx \dfrac{2m_1}{m_2}$ → target moves forward at low velocity

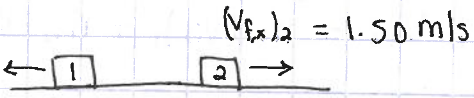
3) massive projectile $(m_1 >> m_2)$

$(v_{f,x})_1 \simeq (v_{i,x})_1$ projectile doesn't change speed (approximately)

$(v_{f,x})_2 \simeq 2(v_{i,x})_1$ target moves forward with about twice speed as projectile

Problem 11.55





- What is speed of 1 just before collision?
  ↘ cons. of energy   system → block + earth.

$$\overset{=0}{K_i} + \overset{=0}{(U_g)_i} + \overset{=0}{(U_{sp})_i} + \overset{=0}{W_{ext}} = K_f + \overset{=0}{(U_g)_f} + \overset{=0}{(U_{sp})_f} + \overset{=0}{\Delta E_{th}}$$

$(U_g)_i = K_f \rightarrow mgy_i = \frac{1}{2}mv_f^2 \rightarrow \boxed{v_f = \sqrt{2gy_i}}$ → Speed of granite cube before collision.

$$(v_{fx})_2 = \frac{2m_1}{m_1+m_2}(v_{i,x})_1$$

$$1.50\text{m/s} = \frac{2m_1}{m_1+m_2}\sqrt{2gy_i} \longrightarrow y_1 = 0.258\text{ m}$$

Momentum in two Dimensions

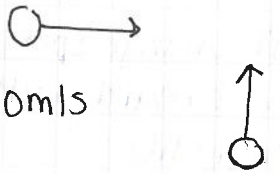
* During a collision in 2 dimensions, each component of momentum is conserved

$\boxed{\vec{P}_{f,x} = \vec{P}_{i,x}}$ and $\boxed{\vec{P}_{f,y} = \vec{P}_{i,y}}$

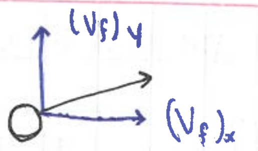
CH 11. + CH 12 April 16, 2019

Problem 11.33

$m_1 = 20g$
$(V_{ix})_1 = 3.0 m/s$



$m_2 = 30g$
$(V_{iy})_2 = 2.0 m/s$



$$\vec{P}_{f,x} = \vec{P}_{i,x}$$

$$(m_1+m_2)\,V_{f,x} = m_1(V_{ix})_1 + m_2\overset{=0}{(V_{i,x})_2}$$

$$V_{f,x} = \frac{m_1(V_{ix})_1}{m_1+m_2} = \frac{(20g)(3.0m/s)}{(20g+30g)} = 1.2\,m/s = V_{f,x}$$

$$\vec{P}_{f,y} = \vec{P}_{i,y}$$

$$(m_1+m_2)\,V_{f,y} = m_1\overset{=0}{(V_{i,x})_1} + m_2(V_{i,y})_2$$

$$V_{f,y} = \frac{m_2(V_{i,y})_2}{m_1+m_2} = \frac{(30g)(2.0m/s)}{(20g+30g)} = 1.2\,m/s = V_{f,y}$$

$$V_f = \sqrt{V_{f,x}^2 + V_{f,y}^2} = \underline{1.7\,m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \underline{45°}$$